PERIODIC COMPOSITE BACKING FILTER FOR THE DESIGN OF ULTRASONIC TRANSDUCERS

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INTRODUCTION

The transducer is the central element of acoustic and ultrasonic detection and characterization technologies [1 - 4]. Some commercial applications of these modalities include ultrasonic nondestructive evaluation NDE, medical diagnostics and undersea imaging.

The piezoelectric element radiates flux of elastic waves in two opposite directions, whereas the transducer performance requires the sonification of a sample material in a single direction. The standard approach is to reduce the elastic flux in the backward wave greatly by LRC impedance choices on the opposite faces. Here a very different approach is suggested, which uses the forbidden gaps in frequency response of certain periodic composite arrays of two elastic materials [5 - 10]. Such backing can completely damp the backward wave rather than to just reduce its magnitude. These forbidden band gaps are more common and wider in the 2-D structures than the 3-D structures which have been investigated to date, so polyethylene cylinders in a square lattice of polycrystalline iron will be studied here. In addition there are new physical effects to be investigated. It is interesting and impressive, that 1 - 3 transducers exist and are being produced [3] well before the pure elastic or pure electromagnetic effects are completely understood.

Several authors, first in [6] and then repeated in [8] have called these arrays "Phononic Band Gap Structures," in analogy to the Photonic Band Gap Structures which were studied and fabricated earlier in electromagnetism. We strongly disagree with this designation, since phonons are 1 - 25 meV quantized lattice waves whereas the scattering process here is due to the macroscopic elastic contrasts $\{\lambda(\vec{r}),\ \mu(\vec{r}),\ 1/\rho(\vec{r})\}$.

The mechanism which creates the bands and the gaps, when they exist, is coherent multiple scattering from the changes in the macroscopic elastic parameters. A wide-band filter is sought here which is the opposite of the case discussed in [7].

THEORY

In Figure 1 a schematic picture of a section of the 2-D structure to be considered is shown and in Fig. 2 the spatial lattice and reciprocal lattices of cylinders in a square lattice are indicated.

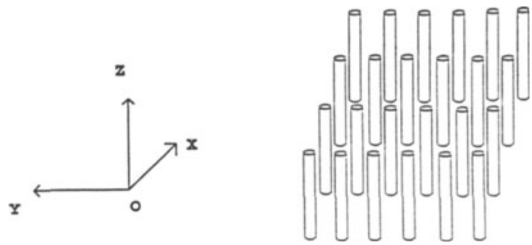

Figure 1. A 3-D schematic of the right circular cylinders of polyethylene in a polycrystalline iron background in a periodic 2-D array. The cylinders form a square lattice, with sides, $L_T = 0.06$ mm for transverse elastic waves, and $L_L = 0.7$ mm for longitudinal elastic waves. The coordinate system is shown.

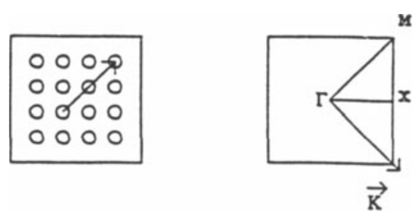

Figure 2 The top view of the direct space lattice with separation $\vec{R}$ between lattice sites on the left. The vector shows the location of the second neighbor. The first Brillouin zone for the square lattice on the right. The high symmetry $\vec{k}$ points are $X(1, 0, 0)$, $\Gamma(0, 0, 0)$, and M $(1, 1, 0)$.

The 3(z) axis is chosen to lie along the symmetry axis of the right circular cylinders of polyethylene in a polycrystalline block of iron. The incident wave, here a backward wave, is along the positive 1(x) axis and the 2(y) axis is chosen to obtain a right handed coordinate system. Both the background Fe material and the polyethylene cylinders are linear, isotropic and homogeneous, LIH separately. Each material depends upon density ρ and Lame parameters $\lambda(c_{11})$ $\mu(c_{44})$ with the displacement amplitude vector given by $\vec{u}(\vec{r})$. For the 2-D stuctures considered in the geometry of Fig 1, $\vec{u} = \vec{u}(x,y)$ and each of $\left\{\lambda, \ \mu, \ 1/\rho\right\}$ denoted as $f_a(\vec{r})$, are periodic with spatial period $\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2$, where (n_1, n_2) are integers,

$$f(\vec{r}) = f(\vec{r} + \vec{R}) \tag{1}$$

where $\vec{a}_1 = (L,\ 0,\ 0)$ and $\vec{a}_2 = (0,\ L,\ 0)$ are the unit vectors for the square lattice in direct space. In eq. (1), f_a represents each of (λ, μ and $1/\rho$) generically. The reciprocal vectors to the lattice of $\vec{R}$ vectors in direct space, can be written as

$$\vec{G} = m_1\vec{g}_1 + m_2\vec{g}_2 \tag{2}$$

where (m_1, m_2) are integers, $\vec{g}_1 = \frac{2\pi}{L}(1,\ 0,\ 0)$ and $\vec{g}_2 = \frac{2\pi}{L}(0,\ 1,\ 0)$ are the basis vectors for the reciprocal lattice.

The equations of elastic waves in LIH media are given by

$$\vec{\nabla} \cdot [(\lambda + 2\mu)(\vec{\nabla} \cdot \vec{u}_L)] = \rho \frac{\partial^2 \vec{u}_L}{\partial t^2} \tag{3}$$

for the longitudinal or compressive wave $\vec{u}_L$ and

$$\vec{\nabla} \cdot [\mu \vec{\nabla} \cdot \vec{u}_T] = \rho \frac{\partial^2 \vec{u}_T}{\partial t^2} \tag{4}$$

for the transverse or shear wave $\vec{u}_T$. The periodicity of the medium parameters $f = \{\lambda,\ \mu,\ 1/\rho\}$ allow their expansion in Fourier series

$$f(\vec{r}) = \sum_{\vec{G}} f_{\vec{G}} e^{i\vec{g}\cdot\vec{r}}, \tag{5}$$

where the summation over $\vec{G}$ runs over all reciprocal vectors. By inverting the Fourier series in eq. (5) and letting f_a denote the medium parameters of the polyethylene "scatterers" and f_b the iron "background" material, the Fourier components $f_{\vec{G}}$ can be written as

$$f_{\vec{G}} = \begin{cases} f_a f_o + f_b(1 - f_o), & \vec{G} = \vec{0} \\ (f_a - f_b) f_o X(\vec{G}), & \vec{G} \neq \vec{0} \end{cases} \tag{6}$$

for each $\vec{G}$. The function $X(\vec{G})$ is the Fourier transform of the support of the f_a circular polyethylene rod which is given by

$$X(\vec{G}) = 2\frac{J_1(|\vec{G}|a)}{(|\vec{G}|a)} \tag{7}$$

and $f_o = \pi a^2 / L^2$ is the filling factor. The common periodicity of all of the f coefficients allow the application of the Floquet - Bloch theorem to the eigensolutions $\vec{u}(\vec{r})$, which must satisfy

$$\vec{u}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}(\vec{r}), \tag{8}$$

where the wave vector $\vec{k}$ is restricted to the first Brillouin zone. Expanding the periodic function $u_{\vec{k}}(\vec{r})$ in the same reciprocal space Fourier series as its coefficient functions as in eq. (5) converts eq. (6) to

$$\vec{u}(\vec{r}) = \sum_{\vec{G}} e^{i(\vec{k}+\vec{G})\cdot\vec{r}} \vec{u}(\vec{k}+\vec{G}). \tag{9}$$

Using eq. (5), (9) in eqs. (3) and (4) the eigenvalue equations becomes

$$\sum_{\vec{G}'}\left\{\sum_{\vec{G}''}\left(\frac{1}{\rho_{\vec{G}-\vec{G}''}}\right)(\lambda+2\mu)_{\vec{G}''-\vec{G}'}(\vec{k}+\vec{G}')(\vec{k}+\vec{G}'')\vec{u}_L(\vec{k}+\vec{G}')\right\}$$

$$= +\omega^2 \vec{u}_L(\vec{k}+\vec{G}), \tag{10}$$

and

$$\sum_{\vec{G}',\vec{G}''}\left(\frac{1}{\rho_{\vec{G}-\vec{G}''}}\right)\mu_{\vec{G}'-\vec{G}''}(\vec{k}+\vec{G}')(\vec{k}+\vec{G}'')\vec{u}_T(\vec{k}+\vec{G}')$$

$$= +\omega^2 \vec{u}_T(\vec{k}+\vec{G}) \tag{11}$$

To discretize eqs. (10), (11), take N_1 $\vec{G}$-vectors and N_2 $\vec{k}$-vectors which reduces these continuuim equations to sets of N_2, N_1 x N_1 matrix equations. This is the plane-wave expansion of eqs. (3), (4). Clearly, dispersion or dissipation could easily be inserted into eqs. (10), (11). Also, these equations can be scaled to $1/L^2$ to provide a frequency response independent of the lattice size at fixed filling factor, density and fixed elastic contrasts. This is clearest from eqs. (3) and (4). For each value of $\vec{k}$, N_1 eigenfrequencies $\omega_{\vec{k}n}$ are found. The label n = 1, 2, 3, ..., N_1 is called the band index and the plot of $\omega_{\vec{k}n}$ vs $\vec{k}$ is called the dispersion of the nth solution. The first 10-15 bands can be calculated accurately with $N_1 = 625$ and $N_2 = 200$ as shown in the next figure. The higher bands are less accurate due to neglected couplings from the discretization. A forbidden gap stop-gap or band-gap is a region of the frequency spectrum $\omega_{\vec{k}}$ in which no eigenfrequency $\omega_{\vec{k}n}$ occurs throughout the first Brillouin zone. The dispersion graph is the values of $\omega_{\vec{k}n_o}$ for fixed n_o= 1, 2, 3, ... as $\vec{k}$ varies from M (1, 1, 0) to Γ(0, 0, 0) to X(1, 0, 0) to M(1, 1, 0). This is called the "band structure" in the electronic physics of crystals. It is essential to have a closed path about the first Brillouin zone if the forbidden gap is to be correctly identified and quantatatively estimated. Some authors are not yet careful enough about this matter, see eg. refs. [5, 8].

Next, the density of states $\rho_D(\omega)$, abbreviated DOS, is defined once one has the dispersion relation $\omega_{\vec{k}n}j$ vs $\vec{k}$. The density of states $\rho_D(\omega)$ is defined such that $\rho_D(\omega)\ d\omega$ is the number of elastic waves with frequency between ω and $\omega +\ d\omega$ and is given by the line integral,

$$\rho_D(\omega) = \frac{A_c}{2\pi^3}\sum_n \int_{\omega=\omega_{\vec{k}n}} dl_\omega \Big/ \left|\nabla_{\vec{k}}\omega_{\vec{k}n}\right| \qquad (12)$$

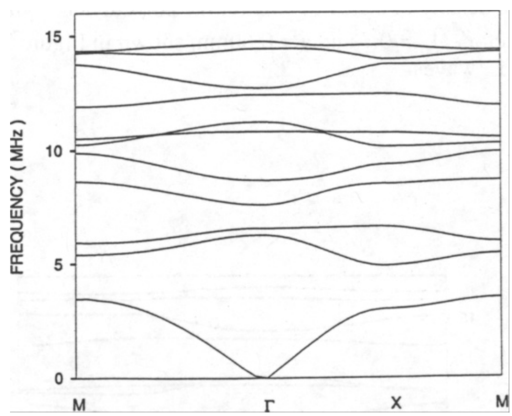

Figure 3. The dispersion relations, $\omega_{\vec{k}n}$ vs $\vec{k}$ for Polyethylene cylinders in a square lattice of iron with filling factor $f_o = 0.7$ for longitudinal elastic waves, $a = 0.60$ mm.

In eq. (12), A_c is the area of the 2-D cell $A_c = |\vec{a}_1 \times \vec{a}_2|$, n is the dispersion curve index and the integration is taken over the surface of constant frequency ω.

The dispersion curves for the longitudinal waves lower bands and their density of states for polyethylene cylinders is polycrystalline iron at filling factor $f_o = 0.7$ are shown in Figs. 3, 4.

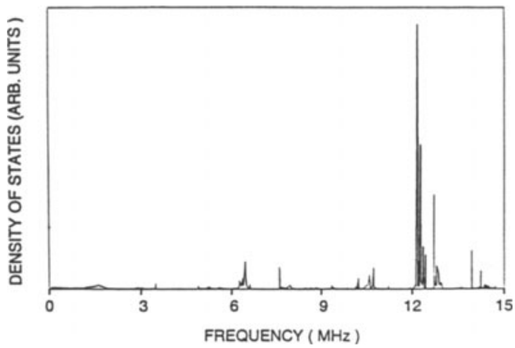

Figure 4. The density of states $\rho_D(\omega)$ for the polyethylene cylinders in iron with filling factor $f_o = 0.7$ and a = 0.60mm shown in Figure 3 for longitudinal, L, waves.

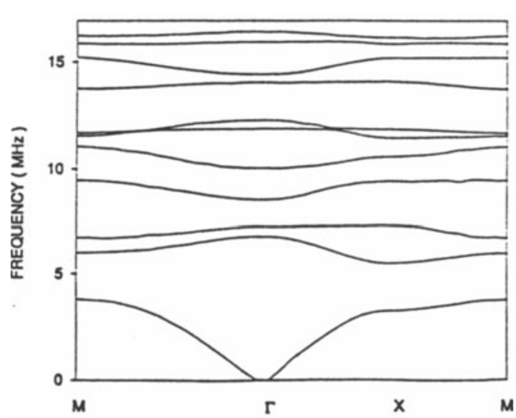

Figure 5. The dispersion relations for polyethylene in iron with $f_o = 0.7$, a = 0.06 mm for shear, T, waves.

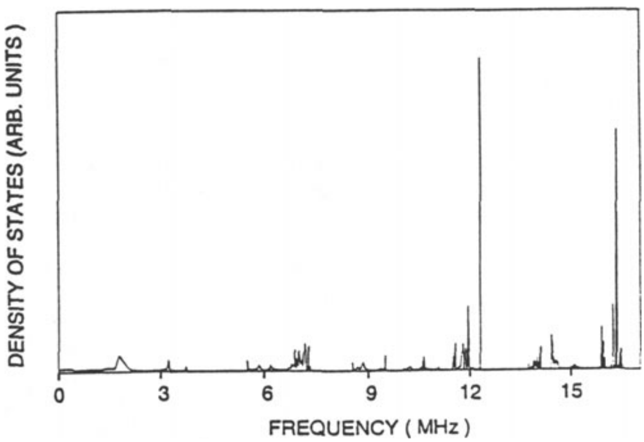

Figure 6. The density of states, $\rho_D(\omega)$, for polyethylene cylinders in iron with f_o = 0.7, a = 0.06 mm for transverse, T, waves.

In Fig. 3, the dispersion curves $\omega_{\vec{k}n}$ vs $\vec{k}$ for the first 11 bands, n = 1, 2, ..., 11, of a wave for a unit cell of 0.7mm. The first forbidden gap for this choice of lattice extends from 3.87 MHz to 6.19 MHz centered at 5 MHz. After the first band, the bands are quite flat, indicative of strong elastic contrast. This is condusive to wide gaps and multiple gaps. The density of states for this filter is presented in Fig. 4. An inspection of eq. (12) shows that the small denominators, due to flat bands, produce the sharp maximum structures which are prominent there.

However, even these wide band-gaps may not be wide enough to filter the spectrum of a transducer with a 5 MHz or 10 MHz central frequency. The following trick [10], can remove this difficulty. Let A, B, C be three of these 2-D structures differing only in their lattice sizes L_A, L_B, L_C. For example, by combining L_A = 0.3 mm and L_C = 0.9 mm with L_B = 0.60 mm in series, one gets a stop-band 1.57 MHz to 8.49 MHz. Similarly, the 10 MHz case can be extended from 2.85 - 16.90 MHz. Similar filters can be designed for any particular frequency range of interest. The opposite case of narrow band filters has been discussed in ref. [8].

The coherent multiple scattering from the 2-D periodic structures studied here should contain new physics of elastic waves. They seem to be a natural for designing new filters especially in series stacks as mentioned in the preceeding section. The defects discussed in [8, 9] also have promise for ultrasonic "lasers" or noncontact sources of elastic waves. There are probably other filters or sources which can be designed and fabricated using these periodic structures. An appeal of NDE is that new physics often generates new applications..

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